Time-varying path following control for port-Hamiltonian systems

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Abstract—This paper is devoted to path following control for port-Hamiltonian systems whose desired path is time-varying. Most of the existing results on path following can only take care of time-invariant paths, hence they cannot be applied to control systems whose environments change, e.g., path following control with moving obstacle avoidance. The proposed method solves this problem by employing decoupling control of three particular directions in the phase space which allows one to assign time-varying potential functions and vector fields.

I. INTRODUCTION

Path following control which is to make the system track the desired path is an important task for control of mechanical systems. Most of the existing results for this problem use the distance between the current state and its desired path. However, it is difficult to measure the smallest distance between the current state and the desired path for complicated desired paths. Therefore several methods not using the distance are proposed to overcome this problem. Salisbury [1] and Hogan [2] proposed a method employing virtual potential function which take its minimum value on the desired path. Li et al. [3], [4] proposed a method called passive velocity field control (PVFC) to design vector fields to track a desired path directly. This method employs a virtual potential energy like function but it does not have intuitive meaning to the control system. Inspired by the idea of PVFC, Duindam et al. [5], [6] proposed a method to design vector fields directly with a natural potential function. We re-formulated the existing results to be applicable to port-Hamiltonian systems and derive a path following method applicable to a wider class of systems. Although the system tracks a fixed desired path in this method, it is not applicable for the desired path crossing itself, e.g., a figure of eight.

In the present paper, the result [7] is further developed to make the system track time-varying desired paths. In this method, the system can track a fixed path crossing itself, such as a figure of eight, by regarding it as a time-varying desired path identical to the fixed path near current state. In order to construct time-varying potential functions and vector fields, we introduce a region moving according to the time-varying desired path. By controlling the system to be included in this region for all time, the system will track the time-varying desired path. Furthermore, if there exist moving obstacles around the desired path, we can avoid collisions between the system and them by choosing the potential energy large near them.

II. PORT-HAMILTONIAN SYSTEMS

Let us consider the following port-Hamiltonian system [8], [9].

\[
\begin{pmatrix}
\dot{q} \\
\dot{p}
\end{pmatrix} =
\begin{pmatrix}
0 & J_{12}(q) \\
-J_{21}(q)^T & J_{22}(q, p)
\end{pmatrix}
\begin{pmatrix}
0 & R_{12}(q) \\
R_{12}(q)^T & R_{22}(q, p)
\end{pmatrix}
\begin{pmatrix}
r \\
\tilde{r}
\end{pmatrix}
\times
\begin{pmatrix}
\frac{\partial H}{\partial q}^T \\
\frac{\partial H}{\partial p}
\end{pmatrix}
+ \begin{pmatrix} 0 \\ G(q) \end{pmatrix} u
\]

\[H(q, p) = \frac{1}{2} p^T M(q)^{-1} p\]

(1)

Here \( x = (q, p) \in \mathbb{R}^l \times \mathbb{R}^m (l \geq m) \). The Hamiltonian function \( H(q, p) \in \mathbb{R} \) describes the kinetic energy of the system. The symmetric semi-positive definite matrix \( R(q, p) \in \mathbb{R}^{(l+m) \times (l+m)} \) describes the energy dissipation. It is supposed that the matrix \( G(q) \in \mathbb{R}^{m \times m} \) is nonsingular, that the matrix \( J_{12}(q) \in \mathbb{R}^{l \times m} \) is full rank such that the matrix \( J_{22}(q, p) \in \mathbb{R}^{m \times m} \) is skew-symmetric, that the matrix \( J(q, p) \in \mathbb{R}^{(l+m) \times (l+m)} \) is skew-symmetric, and that matrices \( J(q, p), R(q, p) \in \mathbb{R}^{m \times m} \) and \( M(q)^{-1} \in \mathbb{R}^{m \times m} \) are continuous. This dynamics is a generalized version of conventional mechanical systems. It can describe a mechanical system with a class of nonholonomic constraints with respect to the velocity \( \dot{q} \). In such a case, the matrices \( J(q, p) \) is determined by the constraints.

In this paper, the inner product on the phase space is defined by

\[\langle p_u, p_v \rangle := p_u^T M(q)^{-1} p_v\]

(2)

for \( p_u, p_v \in \mathbb{R}^m \). Accordingly, the norm is defined by

\[\|p_u\| := \sqrt{\langle p_u, p_u \rangle} \geq 0\]

(3)

A scalar valued function \( \text{sgn}(x) \in \mathbb{R} \) returns the sign of the argument \( x \in \mathbb{R} \) as

\[\text{sgn}(x) := \begin{cases} 
1 & (x \geq 0) \\
-1 & (x < 0)
\end{cases}\]

(4)
III. MAIN RESULTS

This section gives the main result of the present paper path following control of port-Hamiltonian system (1). Here we deal in the desired path characterized by the configuration state $q$ and time $t$. Let us consider a potential function $U_1(q, t)$ which takes its minimum value on the desired path for all time $t$. More precisely, the potential function $U_1(q, t)$ is chosen in such a way that the following assumption holds.

**Assumption 1:** The scalar function $U_1 \in C^1 : \mathbb{R}^l \times \mathbb{R} \to \mathbb{R}$ satisfies the following conditions.
- $U_1(q, t) \geq 0$.
- $U_1(q, t)$ takes its minimum value 0 if and only if $q$ is on the desired path.

We call the value of this potential function $U_1(q, t)$ potential energy. Furthermore, let us define the desired vector $p_w(q, t)$ and the altitudinal vector $p_w(q, t)$ on the phase space.

**Assumption 2:** The vector valued function $p_w \in C^1 : \mathbb{R}^l \times \mathbb{R} \to \mathbb{R}^m$ satisfies the following conditions.
- $\langle (J_{12}(q)^T - R_{12}(q)^T)(\partial U_1/\partial q)^T, p_w(q, t) \rangle = 0$.

**Assumption 3:** The vector valued function $p_{we} \in C^1 : \mathbb{R}^l \times \mathbb{R} \to \mathbb{R}^m$ satisfies the following conditions.
- There exists a scalar $k_1(q, t)$ satisfying $p_{we}(q, t) = k_1(q, t)(J_{12}(q)^T - R_{12}(q)^T)(\partial U_1/\partial q)^T$.

Let $p_{we}(q, t)$ and $p_{we}(q, t)$ denote the normalized version of $p_{we}(q, t)$ and $p_{we}(q, t)$ respectively, when $\parallel p_{we}(q, t) \parallel \neq 0$ and $p_{we}(q, t) \neq 0$, namely,

\[
p_{we}(q, t) := \frac{p_{we}(q, t)}{\parallel p_{we}(q, t) \parallel}, \quad p_{we}(q, t) := \frac{p_{we}(q, t)}{\parallel p_{we}(q, t) \parallel}.
\]

Due to Assumptions 3, $(p_{we}, p_{we}) = 0$.

Next let us decompose $p$ into three elements: one is linearly dependent on $p_{we}(q, t)$, one is linearly dependent on $p_{we}(q, t)$ and the other is orthogonal to $p_{we}(q, t)$ and $p_{we}(q, t)$ which is denoted by $p_{we}(q, p, t)$. That is, $p$ is decomposed as

\[
p = \alpha_\omega(q, p, t)\ p_{we}(q, t) + \alpha_\omega(q, p, t)\ p_{we}(q, t) + \hat{p}(q, p, t),
\]

where

\[
\alpha_\omega(q, p, t) := \langle p_{we}(q, t), p \rangle, \\
\alpha_\omega(q, p, t) := \langle p_{we}(q, t), p \rangle, \\
p_{we}(q, p, t) := p - \alpha_\omega(q, p, t)p_{we}(q, t) - \alpha_\omega(q, p, t)p_{we}(q, t).
\]

According to the decomposition (5), we can decompose the Hamiltonian function as

\[
H(q, p) = H_{k_{\omega}}(q, p) + H_{k,\omega}(q, p) + H_{k,\omega}(q, p, t)
\]

\[
H_{k_{\omega}}(q, p, t) = \frac{1}{2} \alpha_\omega(q, p, t)^2
\]

\[
H_{k,\omega}(q, p, t) = \frac{1}{2} \alpha_\omega(q, p, t)^2
\]

\[
H_{k,\omega}(q, p, t) = \frac{1}{2} \langle p_{we}(q, p, t), p_{we}(q, p, t) \rangle.
\]

Here $H_{k_{\omega}}(q, p, t)$, $H_{k,\omega}(q, p, t)$ and $H_{k,\omega}(q, p, t)$ denote the kinetic energy with respect to the desired direction $p_{we}(q, t)$, that with respect to the altitudinal one $p_{we}(q, t)$ and that with respect to the undesired one $p_{we}(q, p, t)$.

Due to Assumption 1, the system tracks the desired path if and only if the condition $U_1(q, t) = 0$ holds. If $p_{we}(q, t) \neq 0$, then the time derivative of $U_1(q, t)$ along the port-Hamiltonian system (1) is calculated as

\[
\frac{dU_1}{dt} = \frac{\partial U_1}{\partial q} + \frac{\partial U_1}{\partial t}
\]

\[
= \frac{\partial U_1}{\partial q} (J_{12} - R_{12}) M^{-1} p + \frac{\partial U_1}{\partial t}
\]

\[
= \alpha_\omega \langle (J_{12} - R_{12}) \frac{\partial U_1}{\partial q} + p_{we} \rangle + \frac{\partial U_1}{\partial t}.
\]

Therefore if $\alpha_\omega = 0$ and $\partial U_1/\partial t = 0$, then the potential energy $U_1$ does not change. Hence if $U_1 = 0$, $\alpha_\omega = 0$ and $\partial U_1/\partial t = 0$, then the system tracks the desired path.

We will design a controller to make $U_1$ and $\alpha_\omega$ converge to 0 but there are some problems. The system cannot track the desired path when $\partial U_1/\partial t \neq 0$. Moreover $p_{we}$ is not defined when $p_{we} = 0$ and $p_{we}$ is similar. Consequently, let us consider a time-varying region $A(t) \subseteq \mathbb{R}^l$ not including those points.

**Assumption 4:** The region $A(t) \subseteq \mathbb{R}^l$ satisfies the following conditions for all time $t$.
- There exists a $q \in A(t)$ such that $U_1(q, t) = 0$.
- $p_{we}(q, t) \neq 0, p_{we}(q, t) \neq 0, \forall q \in A(t)$.
- $\parallel (J_{12} - R_{12}) (\partial U_1/\partial q)^T \parallel \neq 0, \forall q \in A(t)$ except on the desired path.
- There exists a neighborhood of $q, B(t) \subseteq \mathbb{R}^l$, for all $q$ on the desired path such that $\parallel (\partial U_1/\partial t)/(J_{12} - R_{12}) (\partial U_1/\partial q)^T \parallel = 0, \forall q \in A(t) \cap B(t)$.
- The initial condition $q(0) \in A(t)$.
- $A(t)$ is connected.
- Measure of $A(t)$ is not 0.

The fourth item of Assumption 4 is to prevent the input defined later from diverging, and this condition is satisfied if $\partial U_1/\partial t(q, t) = 0$ for all $q \in A(t)$. We want to design a controller to make $q$ stay in $A(t)$ for all time $t$. For this purpose let us define another potential function $U_2(q, t)$ which becomes large enough near the boundary of $A(t)$.

**Assumption 5:** The scalar function $U_2 \in C^1 : \mathbb{R}^l \times \mathbb{R} \to \mathbb{R}$ satisfies the following conditions for all $q \in A(t)$.
- $U_2(q, t) \geq 0$.
- $U_2(q, t) \gg 1$ near the boundary of $A(t)$.
- There exists a constant $k_2 > 0$ such that if $U_2(q, t) \ll 1$ then $U_2^2/(\parallel (J_{12} - R_{12}) (\partial U_2/\partial q)^T, p_{we} \parallel) \ll 1$.

Due to the second item of Assumption 5, if $U_2(q, t)$ does not diverge, then $q$ is in $A(t)$ for all time $t$. The time derivative of $U_2(q, t)$ along the port-Hamiltonian system (1)
In the third step, a controller called gradient controller is added to make \( U_1(q, t) \) and \( \alpha_w(q, p, t) \) converge to 0 while \( \alpha_w(q, t) \) and \( \| p_w(q, p, t) \| \) do not change. In this control system, since the asymptotic controller reduces \( H_{k,w}(q, p, t) \), \( H_{k,w}(q, p, t) \) and \( U_1(q, t) \) down to 0, the condition \( H(q, p) + U_1(q, t) = H_{k,w}(q, p, t) \) is achieved asymptotically while \( H_{k,w}(q, p, t) \) is time invariant.

In the fourth step, a controller called velocity controller is added to make \( q \) stay in \( A(t) \) for all time \( t \). This controller prevent \( U_2(q, t) \) from diverging by controlling \( \alpha_w \).

### A. Nominal controller

This subsection gives the nominal controller. The objective of this controller is to make \( \alpha_w(q, p, t) \), \( \alpha_w(q, p, t) \) and \( \| p_w(q, p, t) \| \) constant in order to decouple.

**Theorem 1:** (i) Consider a port-Hamiltonian system (1) with a vector \( p_{w}(q, t) \) and \( p_{w}(q, t) \). Suppose that Assumptions 2 and 3 hold. Then \( \alpha(w(q, p), \alpha_w(q, p, t) \) and \( \| p_w(q, p, t) \| \) are constant along the state trajectory of the closed loop system derived by the nominal controller defined by

\[
\begin{align*}
\eta_n &= (p_{we}, p) G^{-1} \left( \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t} \right) \\
&+ (p_{we}, p) G^{-1} \left( \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t} \right) \\
&- \left( \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{we} \right) G^{-1} p_{we} \\
&- \left( \eta_{Jw} + \xi_{Rw} + \frac{\partial p_{we}}{\partial t}, p_{we} \right) G^{-1} p_{we} + G^{-1} \hat{R} M^{-1} p.
\end{align*}
\]

Here \( \eta_{Jw}(q, p, t) \) and \( \eta_{Jw}(q, p, t) \) are defined by

\[
\eta_{Jw}(q, p, t) := \xi_{Jw}(q, p, t) - \hat{J}(q, p) M^{-1} p_{we},
\]

skew-symmetric matrix \( \hat{J}(q, p) \in \mathbb{R}^{(l+m)\times(l+m)} \) and symmetric matrix \( \hat{R}(q, p) \in \mathbb{R}^{(l+m)\times(l+m)} \) are defined by

\[
\begin{align*}
\hat{J} &:= \frac{1}{2} M \begin{pmatrix} \\
\frac{\partial M^{-1}}{\partial q_1} & \cdots & \frac{\partial M^{-1}}{\partial q_l} \\
\frac{\partial M^{-1}}{\partial q_{l+1}} & \cdots & \frac{\partial M^{-1}}{\partial q_{2l}} \\
\frac{\partial M^{-1}}{\partial q_{2l+1}} & \cdots & \frac{\partial M^{-1}}{\partial q_{3l}} \\
\end{pmatrix} L \\
\hat{R} &:= \frac{1}{2} M \begin{pmatrix} \\
\frac{\partial M^{-1}}{\partial q_1} & \cdots & \frac{\partial M^{-1}}{\partial q_l} \\
\frac{\partial M^{-1}}{\partial q_{l+1}} & \cdots & \frac{\partial M^{-1}}{\partial q_{2l}} \\
\frac{\partial M^{-1}}{\partial q_{2l+1}} & \cdots & \frac{\partial M^{-1}}{\partial q_{3l}} \\
\end{pmatrix} L,
\end{align*}
\]

and \( \xi_{Jw}(q, p, t), \xi_{Rw}(q, p, t) \) and \( \xi_{Jw}(q, p, t) \) are constant in order to decouple.
\( \mathbb{R}^m \) are vectors defined by

\[
\begin{align*}
\xi_{jw} & := \left( \frac{\partial p_{uw}}{\partial q} + \frac{1}{2} M \left[ \frac{\partial M^{-1}}{\partial q} p_{uw} \cdots \frac{\partial M^{-1}}{\partial q} p_{uw} \right] \right) J_l M^{-1} p \\
\xi_{Rw} & := - \left( \frac{\partial p_{uw}}{\partial q} + \frac{1}{2} M \left[ \frac{\partial M^{-1}}{\partial q} p_{uw} \cdots \frac{\partial M^{-1}}{\partial q} p_{uw} \right] \right) K_l M^{-1} p \\
\xi_{Jw} & := \left( \frac{\partial \bar{p}_{uc}}{\partial q} + \frac{1}{2} M \left[ \frac{\partial M^{-1}}{\partial q} \bar{p}_{uc} \cdots \frac{\partial M^{-1}}{\partial q} \bar{p}_{uc} \right] \right) J_l M^{-1} p \\
\xi_{Rw} & := - \left( \frac{\partial \bar{p}_{uc}}{\partial q} + \frac{1}{2} M \left[ \frac{\partial M^{-1}}{\partial q} \bar{p}_{uc} \cdots \frac{\partial M^{-1}}{\partial q} \bar{p}_{uc} \right] \right) K_l M^{-1} p.
\end{align*}
\]

(ii) \( \xi_{jw}, \xi_{Rw}, \xi_{Jw}, \xi_{Rw}, \eta_{jw} \) and \( \eta_{Jw} \) satisfy the following equation

\[
\begin{align}
\langle \xi_{jw}, p_{uw} \rangle & = \langle \xi_{Rw}, p_{uw} \rangle = \langle \xi_{Jw}, p_{uw} \rangle = \langle \xi_{Rw}, p_{uw} \rangle = 0 \quad \text{(11)} \\
\langle \eta_{jw}, p_{uw} \rangle & = \langle \eta_{Jw}, p_{uw} \rangle = 0.
\end{align}
\]

**Proof:** We prove only (i) due to space limitation. Let us consider a feedback system with the port-Hamiltonian system (1) with the feedback \( u = u_n + \bar{u} \). First, we prove that \( \alpha_{uw} \) is constant. The time derivative of \( \alpha_{uw} \) is calculated as

\[
\begin{align*}
\frac{d\alpha_{uw}}{dt} & = \langle G u - \bar{R} M^{-1} p, p_{uw} \rangle + \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p \rangle \\
& \quad - \langle p_{uw}, p \rangle \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p_{uw} \rangle \\
& = \langle p_{uw}, p \rangle \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p_{uw} \rangle \\
& = 0.
\end{align*}
\]

This proves that \( \alpha_{uw} \) is constant.

First, we prove that \( \alpha_{\tilde{w}} \) is constant. The time derivative of \( \alpha_{\tilde{w}} \) is calculated as

\[
\begin{align*}
\frac{d\alpha_{\tilde{w}}}{dt} & = \langle G u - \bar{R} M^{-1} p, p_{uw} \rangle + \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p \rangle \\
& \quad - \langle p_{uw}, p \rangle \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p_{uw} \rangle \\
& = 0.
\end{align*}
\]

This proves that \( \alpha_{\tilde{w}} \) is constant.

Next we prove that \( \| \tilde{p} \| \) is constant. In order to prove that \( \tilde{p} = \sqrt{H_{k,\tilde{w}}} \) is constant. The time derivative of \( H_{k,\tilde{w}} \) along the closed loop system is calculated as

\[
\begin{align*}
\frac{dH_{k,\tilde{w}}}{dt} & = \langle G u - \bar{R} M^{-1} p, p_{uw} \rangle \\
& \quad - \langle p_{uw}, p \rangle \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p \rangle \\
& \quad - \langle p_{uw}, p \rangle \langle \eta_{jw} + \xi_{Rw} + \frac{\partial p_{uw}}{\partial t}, p_{uw} \rangle \\
& = 0.
\end{align*}
\]

which implies that \( H_{k,\tilde{w}} \) is constant. This also suggests that \( \tilde{p} = \sqrt{H_{k,\tilde{w}}} \) is constant as well. This completes the proof.

Let us consider a feedback system with the port-Hamiltonian system (1) with the feedback \( u = u_n + \bar{u} \). The time derivative of \( \alpha_{uw}, \alpha_{\tilde{w}} \) and \( H_{k,\tilde{w}} \) along the closed loop system are calculated as

\[
\begin{align*}
\frac{d\alpha_{uw}}{dt} & = \langle G \bar{u}, p_{uw} \rangle \quad \text{(15)} \\
\frac{d\alpha_{\tilde{w}}}{dt} & = \langle G \bar{u}, p_{uw} \rangle \quad \text{(16)} \\
\frac{dH_{k,\tilde{w}}}{dt} & = \langle G \bar{u}, p_{uw} \rangle. \quad \text{(17)}
\end{align*}
\]

If we choose \( \bar{u} = k_3 G^{-1} p_{uw} \) with a scalar function \( k_3(q,p,t) \), then \( \bar{u} \) changes \( \alpha_{uw} \) without changing \( \alpha_{\tilde{w}} \) and \( H_{k,\tilde{w}} \) since \( \langle p_{uw}, p_{uw} \rangle = \langle p_{uw}, p_{uw} \rangle = 0 \). In the same way, we can change \( \alpha_{\tilde{w}} \) and \( H_{k,\tilde{w}} \) without changing others.

**B. Path following**

Let us introduce the asymptotic controller. The objective of this controller is to make \( p_{uw}(q, p, t) \) converge to 0. Let us define the asymptotic controller by

\[
u_{\alpha} = -\beta_3 G^{-1} p_{\tilde{w}}
\]

where a continuous function \( \beta_3(q,p,t) > 0 \in \mathbb{R} \) is a design parameter.

Next let us introduce the gradient controller. The objective of this controller is to reduce the potential function \( U_1 \) to achieve the path following control. If \( q \in A(t) \) then the time derivative of \( U_1(q,t) \) along the port-Hamiltonian system (1) is calculated as

\[
\begin{align*}
\frac{dU_1}{dt} & = \langle d_q U_1, p \rangle + \frac{dU_1}{dt} = \alpha_{\tilde{w}} \langle d_q U_1, p_{\tilde{w}} \rangle + \frac{dU_1}{dt}. \quad \text{(19)}
\end{align*}
\]

The gradient controller controls \( \alpha_{\tilde{w}} \) to reduce the potential function \( U_1 \). Let us define the desired value \( \alpha_{\tilde{w}}(q,t) \in \mathbb{R} \) of \( \alpha_{\tilde{w}} \) that if \( \alpha_{\tilde{w}} = \alpha_{\tilde{w}} \) then \( U_1 \) decreases. The gradient controller makes \( \alpha_{\tilde{w}} \) converge to \( \alpha_{\tilde{w}} \). More precisely, the desired value \( \alpha_{\tilde{w}}(q,t) \) is chosen in such a way that the following assumption holds.

**Assumption 6:** The desired value \( \alpha_{\tilde{w}} \in C^1 : \mathbb{R}^l \times \mathbb{R} \rightarrow \mathbb{R} \) satisfies the following conditions for \( (q,t) \in A(t) \times \mathbb{R} \):

- \( \alpha_{\tilde{w}}(q,t) = 0 \) if \( U_1(q,t) = 0 \).
- \( \alpha_{\tilde{w}}(q,t) \text{sgn} (d_q U_1, p_{\tilde{w}}) < -\langle \partial U_1/\partial t \rangle/\|d_q U_1\| \) if \( U_1(q,t) > 0 \).

For example, the following \( \alpha_{\tilde{w}}(q,t) \) satisfies this assumption.

\[
\alpha_{\tilde{w}} = \begin{cases} 0 & \text{when } U_1 = 0 \\
\gamma_p + \frac{\partial p_{\tilde{w}}}{\|d_q U_1\|} \text{sgn} (d_q U_1, p_{\tilde{w}}) & \text{when } U_1 > 0
\end{cases}
\]

Here a continuous scalar function \( \gamma_p(q,t) \geq 0 \in C^1 \) is design parameter which takes its minimum value 0 if and only if \( U_1(q,t) = 0 \).

Let us define the gradient controller by

\[
u_p = -\beta_3 (\alpha_{\tilde{w}} - \alpha_{\tilde{w}}) G^{-1} p_{\tilde{w}} + \frac{d\alpha_{\tilde{w}}}{dt} \frac{G^{-1} p_{\tilde{w}} - d_q U_1}{dt}
\]

where a continuous function \( \beta_3(q,p,t) > 0 \in \mathbb{R} \) is a design parameter. \( d\alpha_{\tilde{w}}/dt(q,p,t) \) in Equation (21) denotes the
The time derivative of $\alpha_{w r}$ along the port-Hamiltonian system (1) calculated as

$$\frac{d\alpha_{w r}}{dt} = - \left\{ \left( J_{12}^T - R_{12}^T \right) \frac{\partial \alpha_{w r}}{\partial q} \cdot p + \frac{\partial \alpha_{w r}}{\partial t} \right\} \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r
In order to prove the convergence of the state to the desired path. In order to prove the convergence of the state to the desired path, it is proven that $U_1$ and $\alpha_{\bar{w}}$ converge to 0. Let us define a Lyapunov like function $V_2(q,p,t) = U_1(q,t) + \frac{1}{2}(\alpha_{\bar{w}}(q,p,t) - \alpha_{\bar{w}}(q,t))^2 \geq 0$.

Since $q \in A(t)$ for $t$, the time derivative of $V_2$ is calculated as:

$$\frac{dV_2}{dt} = \frac{dU_1}{dt} + (\alpha_{\bar{w}} - \alpha_{\bar{w}})\left(\frac{d\alpha_{\bar{w}}}{dt} - \frac{d\alpha_{\bar{w}}}{dt}\right)$$

$$= \alpha_{\bar{w}}(dU_1, p_{\bar{w}}) + \frac{dU_1}{dt} + (\alpha_{\bar{w}} - \alpha_{\bar{w}})\left(\langle Gu - \bar{R}M^{-1}p, p_{\bar{w}}\rangle\right)$$
$$\frac{d\alpha_{\bar{w}}}{dt}$$

For each element $k$, Assumptions 1, 2 and 3 do not need to hold for $\alpha_{\bar{w}}$, $\alpha_{\bar{w}}$ and $p_{\bar{w}}$ related the potential function $U_1$, then these elements are controlled independently. The system converges to the desired path and avoids collisions with moving obstacles by controlling the altitudinal element $\alpha_{\bar{w}}p_{\bar{w}}$ of $p$. Furthermore, $q$ stays in the region $A(t)$ for all time $t$ by controlling the desired element $\alpha_{\bar{w}}p_{\bar{w}}$ of $p$.

Since $q$ stays in the region $A(t)$ for all time $t$, Assumptions 1, 2 and 3 do not need to hold for $q \notin A(t)$.

**IV. CONCLUSIONS**

This paper is devoted to a new path following method whose desired path is time-varying for a class of port-Hamiltonian systems. In the proposed method systems can avoid collisions with moving obstacles. We design potential functions and vector fields as time-varying functions. By controlling the desired, the altitudinal and the undesired phase states independently, we can derive a control method to make systems converge to the time-varying desired path. Furthermore by making the potential energy large near the obstacles, we can avoid collisions between the systems and the obstacles.

**REFERENCES**


