Passivity based control and learning control of Hamiltonian systems 2

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Outline

1. What is learning control?
2. Variational symmetry and learning
3. Applications
4. Conclusion
What is learning control? (1/4)

Iterative Learning Control (ILC) and Iterative Feedback Tuning (IFT)

ILC adjusts FF input and IFT adjusts FB controller by learning!
Learning control optimizes the parameters by steepest decent method.
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Find an optimal $x \in X$ minimizing the cost function $\Gamma : X \rightarrow \mathbb{R}$. 
Learning control optimizes the parameters by steepest decent method.

Find an optimal \( x \in X \) minimizing the cost function \( \Gamma : X \rightarrow \mathbb{R} \).

Steepest decent method change the parameter \( x \) in the direction of gradient of \( \Gamma(x) \).

\[
    x_{(k+1)} = x_{(k)} - K \cdot \nabla \Gamma(x)
\]
Then, what is a problem?
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The cost function $\Gamma(x)$ is unknown!
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- The cost function $\Gamma(x)$ is unknown!

(Ex) To find a feedback gain $K$:

\[
\begin{aligned}
\dot{x} &= Ax + Bu \\
u &= Kx
\end{aligned}
\]

\[
\Gamma(K) := \int_{0}^{T} x^{T}Qx + u^{T}Rudt, \quad x(0) = x^{0}
\]

If the system parameters $A$ and $B$ are unknown, then $\Gamma(\cdot)$ is unknown as well.
Then, what is a problem?

The cost function $\Gamma(x)$ is unknown!

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$$

If the system parameters $A$ and $B$ are unknown, then $\Gamma(\cdot)$ is unknown as well.

$\rightarrow$ Estimate (learn) $\nabla \Gamma(x)$ by experiments

(Ex) In case of $x \in \mathbb{R}$...

$$
\nabla \Gamma(x) \approx \frac{\Gamma(x + \Delta x) - \Gamma(x)}{\Delta x}
$$
What is learning control? (4/4)

- Again, what is a problem?
- Space $X(\ni x)$ can be high or infinite dimensional
Again, what is a problem?

Space $X(\ni x)$ can be high or infinite dimensional.

(Ex) To find a FF input $u$:

$$\dot{x} = Ax + Bu$$

$$\Gamma(u) := \int_0^T x^T Q x + u^T R u \, dt, \quad x(0) = x^0$$
What is learning control? (4/4)

Again, what is a problem?

- Space $X(\ni x)$ can be high or infinite dimensional

(Ex) To find a FF input $u$:

$$\dot{x} = Ax + Bu$$

$$\Gamma(u) := \int_0^T x^T Q x + u^T R u dt, \ x(0) = x^0$$

- Need to estimate the gradient!

(Ex) The following procedure needs infinite experiments!

$$\nabla \Gamma(x) \approx \frac{\Gamma(x + \Delta x) - \Gamma(x)}{\Delta x}$$
Gradient and variational adjoint (1/2)

Problem setting

- Plant: $\Sigma : U \rightarrow Y$

(Ex)

\[
\begin{aligned}
y &= \Sigma(u) : \quad \begin{cases} 
\dot{x} &= f(x, u) \\
y &= h(x, u) 
\end{cases} 
\end{aligned}
\]

- Cost function: $\tilde{\Gamma} : U \times Y \rightarrow \mathbb{R}$

(Ex)

\[
\tilde{\Gamma}(u, y) := \int_0^T y^T Q y + u^T R u \, dt
\]
Gradient and variational adjoint (1/2)

Problem setting

Plant: $\Sigma : U \rightarrow Y$

(Ex)

$y = \Sigma(u) : \begin{cases} \dot{x} &= f(x, u) \\ y &= h(x, u) \end{cases}$

Cost function: $\tilde{\Gamma} : U \times Y \rightarrow \mathbb{R}$

(Ex)

$\tilde{\Gamma}(u, y) := \int_{0}^{T} y^T Q y + u^T R u \, dt$

Cost reduces to $\Gamma(u) := \tilde{\Gamma}(u, \Sigma(u))$.

$\tilde{\Gamma}(u, y)$ are known

$\Sigma(u)$ can be measured by experiments
Gradient and variational adjoint (2/2)

Gradient $\nabla \Gamma(u)$:

$$\langle \nabla \Gamma(u), du \rangle = \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle \nabla_y \tilde{\Gamma}(u, \Sigma(u)), dy \rangle$$
Gradient and variational adjoint (2/2)

Gradient $\nabla \Gamma(u)$:

\[
\langle \nabla \Gamma(u), du \rangle = \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle \nabla_y \tilde{\Gamma}(u, \Sigma(u)), dy \rangle \\
= \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle \nabla_y \tilde{\Gamma}(u, \Sigma(u)), d\Sigma(u)(du) \rangle
\]
Gradient and variational adjoint (2/2)

Gradient $\nabla \Gamma(u)$:

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\langle \nabla \Gamma(u), du \rangle = \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle \nabla_y \tilde{\Gamma}(u, \Sigma(u)), dy \rangle
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$$
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$$

$$
= \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle (d\Sigma(u))^* \nabla_y \tilde{\Gamma}(u, \Sigma(u)), du \rangle
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Gradient and variational adjoint (2/2)

Gradient $\nabla \Gamma(u)$:

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$$\nabla \Gamma(u)$$

Information of the variational adjoint $(d\Sigma(u))$ is needed for learning control!
Gradient $\nabla \Gamma(u)$:

$$\langle \nabla \Gamma(u), du \rangle = \langle \nabla_u \tilde{\Gamma}(u, \Sigma(u)), du \rangle + \langle \nabla_y \tilde{\Gamma}(u, \Sigma(u)), dy \rangle$$

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$$\nabla \Gamma(u)$$

Information of the variational adjoint $(d\Sigma(u))^*$ is needed for learning control!
Learning in control engineering

- IFT (Hjalmarsson et al.)
  - Estimation of finite number of parameters
  
  \[ \nabla \Gamma(x) \approx \frac{\Gamma(x + \Delta x) - \Gamma(x)}{\Delta x} \]

- ILC (Arimoto et al.)
  - Estimate \((d\Sigma(u))\)* by direct feedthrough term
  - Special cost function for trajectory tracking
  - Simple learning law

- ILC (proposed)
  - Estimate \((d\Sigma(u))\)* by experiments
  - Only for Hamiltonian control systems
  - Applicable to several optimal control problems
Outline

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Hamiltonian control system with dissipation

\[ y = \Sigma(u) : \begin{cases} 
\dot{x} &= (J - R) \frac{\partial H(x, u, t)^T}{\partial x} \\
y &= -\frac{\partial H(x, u, t)^T}{\partial u}
\end{cases} \]
ILC

Let $u(i)$, $y(i)$ and $x(i)$ denote input, output and the state of the $i$-th experimental data of the time interval $t \in [t^0, t^1]$.

Update the input $u(i+1)$ such that it converges to an optimal value.
Variational symmetry and gradient

Gradient

\[ \nabla \Gamma(u, \Sigma(u)) = \nabla_u \Gamma(u, y) + (d\Sigma(u))^* \nabla_y \Gamma(u, y) \]

Variational symmetry

The dynamics of \((d\Sigma(u))^*\) coincides with the time-reversal dynamics of \(d\Sigma(\bar{u})\)!

Derivative can be approximated by difference:

\[ d\Sigma(u)(v) \approx \Sigma(u + v) - \Sigma(u) \]
Cost functions

- Cost function $\Gamma(x(t^1), y, u)$
  - Trajectory tracking:
    \[
    \Gamma = \int_{t_0}^{t_1} \|y(t) - y^d(t)\|^2 \, dt
    \]
  - Trajectory generation (Optimal control):
    \[
    \Gamma = \|x(t^1) - x^{1d}\|^2 + \gamma_u \int_{t_0}^{t_1} \|u(t)\|^2 \, dt
    \]
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Trajectory tracking control (1/2)
Experimental system
Experimental result
Input saturation is characterized by a saturation function $\phi(\cdot)$.
Simulation with saturation

Without considering input saturation

[Graphs showing simulations with and without input saturation]
Simulation with saturation

- Without considering input saturation

- With considering input saturation (50 [Nm])
**Application to nonholonomic systems**

- Applicability: $J, R$ have to be constant
- Not applicable to nonholonomic systems
- Let the system satisfy the condition via feedback
Construct a feedback only using the information of constraints
ILC and IFT

IFT adjusts FB controller while ILC adjusts FF input.
Application to jumping robots (1/3)

Jumping Robot [Hyon’04]

This robot has **passive running gait**!

Application of ILC to this robot with $y = \theta$
Due to the symmetry of the passive gates, the following cost function is adopted.

\[
\Gamma(u, y; \dot{y}; x^0) = k_1 \|u\|_{L^2}^2 + k_2 \|y + \mathcal{R}(y)\|_{L^2}^2 + k_3 \|\dot{y} - \mathcal{R}(\dot{y})\|_{L^2}^2
\]

where \( \mathcal{R} \) is the time-reversal operator:

\[\mathcal{R}(u)(t) = u(t^1 - t).\]

Time-derivative of \( \dot{y} \) can be evaluated.
Application to jumping robots (3/3)

![Graphs showing time versus amplitude and cost function](image)

- **θ** in the last iteration before learning
- **D** in the last iteration before learning
- Cost function
Feedback system is constructed by GCT.

Suppose that Hamiltonian function depends on the parameter $k \in \mathbb{R}^m$

\[
\begin{align*}
\dot{x} &= (J - R) \frac{\partial H(x, t, k)}{\partial x}^T \\
z &= -\int_{t_0}^{t_1} \frac{\partial H(x, t, k)}{\partial k}^T \, dt
\end{align*}
\]

Then the map $k \mapsto z$ has variational symmetry!

Any function(-al) of $k$ and $z$ can be minimized by experimental data!

Although ordinary IFT needs $m + 1$ experiments for one step iteration, the proposed method requires only 2 experiments!
For example, the Hamiltonian function of mass-spring-damper system becomes

\[
H(q, p) = \frac{1}{2m} \|p\|^2 + \frac{k}{2} \|q\|^2
\]

\[
z = -\int_{t_0}^{t_1} \|q\|^2 dt
\]

with the spring coefficient \(k\).

E.g., the following (optimal control type) cost function can be minimized:

\[
\Gamma(k, z) = \int_{t_0}^{t_1} \left( \|q\|^2 + \gamma \| kq \|^2 \right) dt
\]
Summary

Gradient of the cost function

\[ \nabla \Gamma(u, \Sigma(u)) = \nabla_u \Gamma(u, y) + (d\Sigma(u))^* \nabla_y \Gamma(u, y) \]

- Variational symmetry of Hamiltonian systems is quite useful.
- ILC for optimal control is proposed based on it.
- Future work
  - Combination with statistical approach (Independent component analysis, Reinforcement learning)
  - Further applications
Conclusion

- Control of electro-mechanical systems
  - Feedback control
    - Passivity based control
  - Feedforward control
    - Optimal control via learning

Future work

Application to real world plants
Application to real world problems
Conclusion

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  - Feedback control
    - Passivity based control
  - Feedforward control
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- Future work
  - Application to real world plants
  - Application to real world problems
Thank you for your attention!

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